Combining KLE based global parameterization and IEnKF for reservoir history matching

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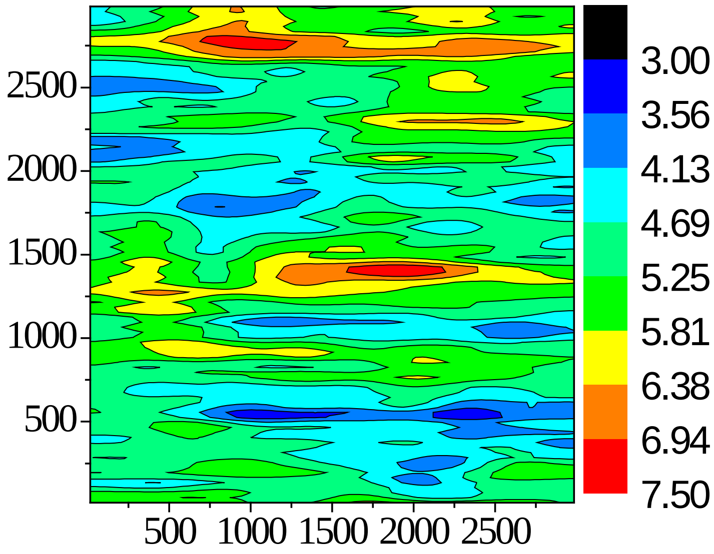
Motivation

Reservoir fields:



Motivation

- Preserving the geostatistical characteristics of the parameter field in the history matching process;
- Taking account of the uncertainty about the correlation parameter, such as anisotropy direction and correlation length.

Karhunen-Loeve expansion

- Gaussian random field $Y(\boldsymbol{x},\omega)$
- Given covariance from prior geology:

$$C_Y(\boldsymbol{x}_1,\boldsymbol{x}_2)=\left\langle Y^{'}(\boldsymbol{x}_1,\omega)Y^{'}(\boldsymbol{x}_2,\omega)\right\rangle$$

- Karhunen-Loeve expansion (KLE):

$$C_Y(\boldsymbol{x}_1,\boldsymbol{x}_2)=\sum_{n=1}^{\infty}\lambda_n f_n(\boldsymbol{x}_1) f_n(\boldsymbol{x}_2)$$

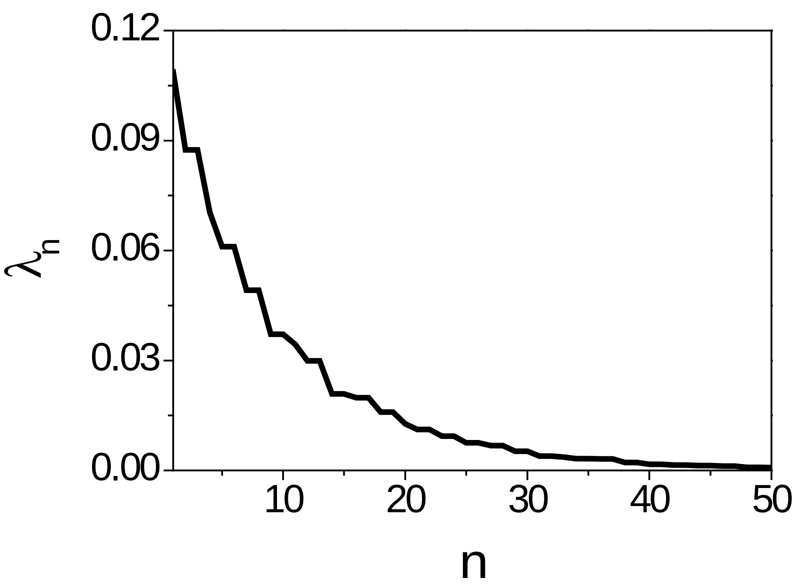
$$Y(\boldsymbol{x},\omega)=\overline{Y}(\boldsymbol{x})+\sum_{n=1}^{\infty}\sqrt{\lambda_n}\, f_n(\boldsymbol{x})\xi_n(\omega)$$

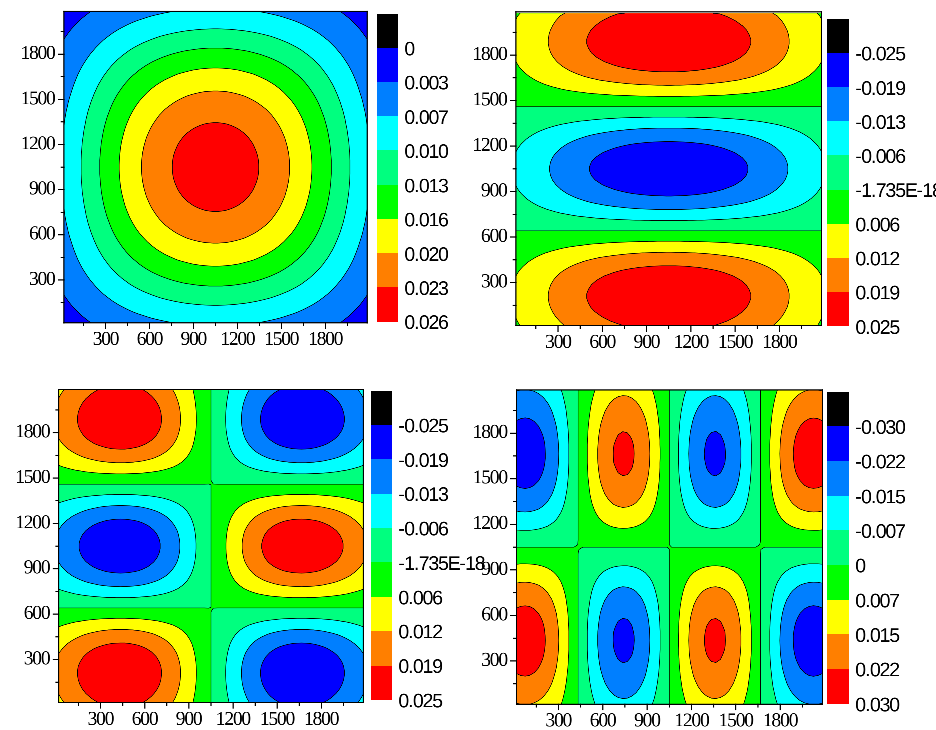
- Deterministic eigenvalues and eigenfuctions: λ_n and $f_n(\boldsymbol{x})$
- Orthogonal Gaussian random variables: $\xi_n(\omega)$

$$\left\langle \xi_n(\omega)\right\rangle = 0, \quad \left\langle \xi_n(\omega)\xi_m(\omega)\right\rangle = \delta_{nm}$$

Karhunen-Loeve expansion

For $C_Y(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sigma_Y^2 \exp\left[-\left(\frac{|x_1 - x_2|}{\eta_x} \right)^2 - \left(\frac{|y_1 - y_2|}{\eta_y} \right)^2 \right].$





$$\sum_{n=1}^{\infty} \lambda_n = Vol(D)\sigma_Y^2$$

$$Y'(\boldsymbol{x}, \omega) = \sum_{n=1}^{N} \xi_n(\omega) \sqrt{\lambda_n} f_n(\boldsymbol{x})$$

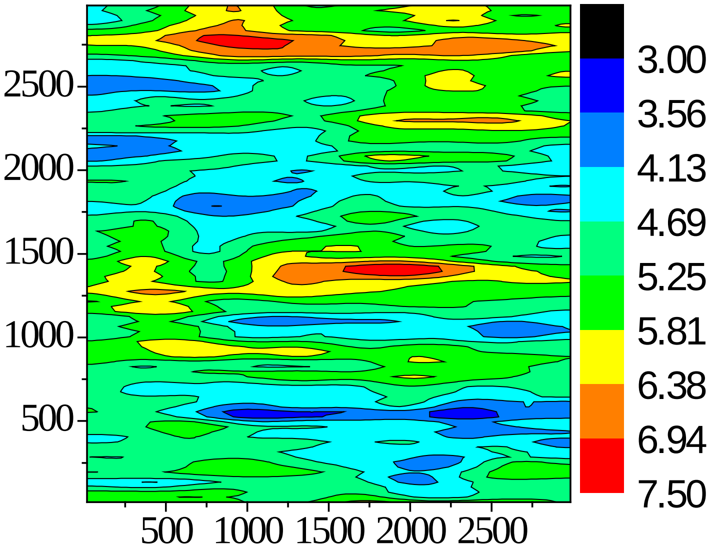
Global Parameterization

- Gaussian random field $Y(\boldsymbol{x}, \omega)$
- KL expansion:

$$Y(\boldsymbol{x}, \omega) = \bar{Y}(\boldsymbol{x}) + \sum_{n=1}^{N} \sqrt{\lambda_n} f_n(\boldsymbol{x}) \xi_n(\omega)$$

- Model parameter:

$$\boldsymbol{m} = \left[\xi_1, \xi_2, \ldots, \xi_N \right]^T$$



Global Parameterization

- Facies field with one embedded facies $p(\boldsymbol{x}, \omega)$
- Level set parameterization:

$$p(\boldsymbol{x}) = p_f H(\Phi(\boldsymbol{x})) + p_n(1 - H(\Phi(\boldsymbol{x}))),$$

$$\Phi(\boldsymbol{x}) = Y(\boldsymbol{x}) - a.$$

- KL expansion:

$$Y(\boldsymbol{x}, \omega) = \overline{Y}(\boldsymbol{x}) + \sum_{n=1}^{N} \sqrt{\lambda_n} f_n(\boldsymbol{x}) \xi_n(\omega)$$

- Model parameter

$$\boldsymbol{m} = \left[\xi_1, \xi_2, \ldots, \xi_N\right]^T$$

- Multi-facies

$$\boldsymbol{m} = \left[\boldsymbol{\xi}_n^1, \boldsymbol{\xi}_n^2, \ldots, \boldsymbol{\xi}_n^{n_f}\right]^T \text{ where } \boldsymbol{\xi}_n^i = (\xi_1^i, \xi_2^i, \ldots, \xi_{N^i}^i)$$

History matching method

- LM_EnRML (Chen and Oliver 2013)

$$\delta \boldsymbol{m}_{l,j} \approx -C_{sc}^{1/2} \Delta \boldsymbol{m}_l^e \left[(1+\lambda_l) I_e + \Delta \boldsymbol{d}_l^{e\mathrm{T}} \Delta \boldsymbol{d}_l^e \right]^{-1}$$

$$\times \Delta \boldsymbol{m}_l^{e\mathrm{T}} \left(\Delta \boldsymbol{m}_{pr}^{e\ -\mathrm{T}} \Delta \boldsymbol{m}_{pr}^{e\ -1} \right) C_{sc}^{-1/2} \left(\boldsymbol{m}_{l,j} - \boldsymbol{m}_{pr,j} \right)$$

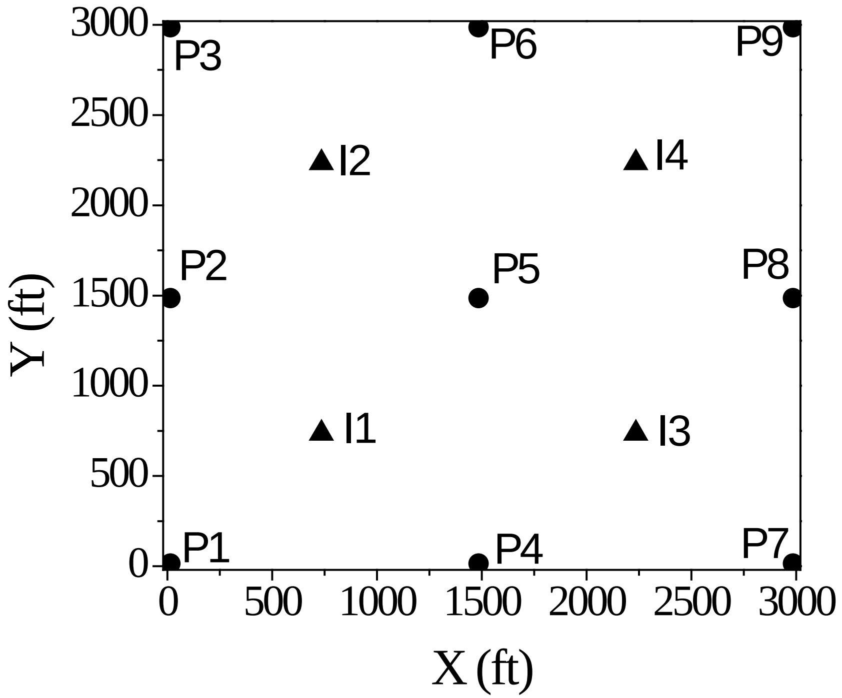
$$- C_{sc}^{1/2} \Delta \boldsymbol{m}_l^e \Delta \boldsymbol{d}_l^{e\mathrm{T}} \left[(1+\lambda_l) I_D + \Delta \boldsymbol{d}_l^e \Delta \boldsymbol{d}_l^{e\mathrm{T}} \right]^{-1}$$

$$\times C_D^{-1/2} \left(g(\boldsymbol{m}_{l,j}) - \boldsymbol{d}_{obs,j} \right), \quad j = 1, \ldots, N_e$$

In this work, we use LM_EnRML as a sequential iterative ensemble filter.

The simulation model

- Model size: 3000ft × 3000ft × 30ft
- Grid: 100×100×1
- Thirteen wells
- History matching period:17 months
- Data: OPR, WCT, BHP



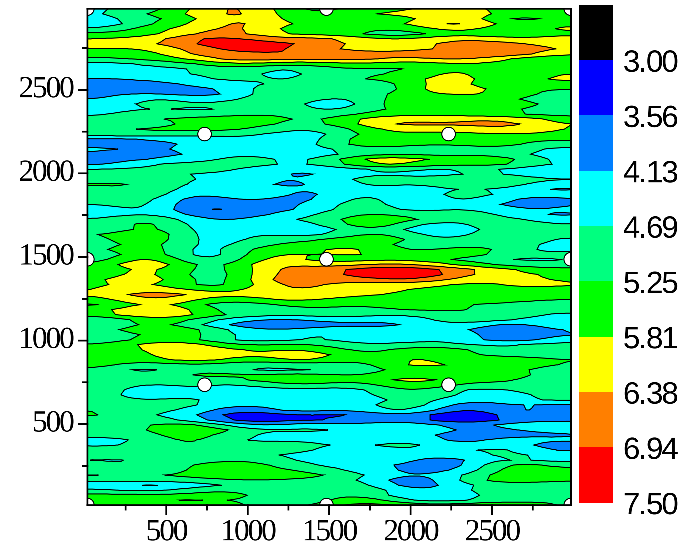
Case 1: Gaussian field estimation

Prior information:

- $\langle \ln k \rangle = 5.0 + \ln \text{mD}$
- $\sigma^2_{\ln k} = 0.5$
- Covariance model:

$$C_{\ln k}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sigma^2_{\ln k} \exp\left[-\frac{|x_1 - x_2|}{\eta_x} - \frac{|y_1 - y_2|}{\eta_y} \right],$$

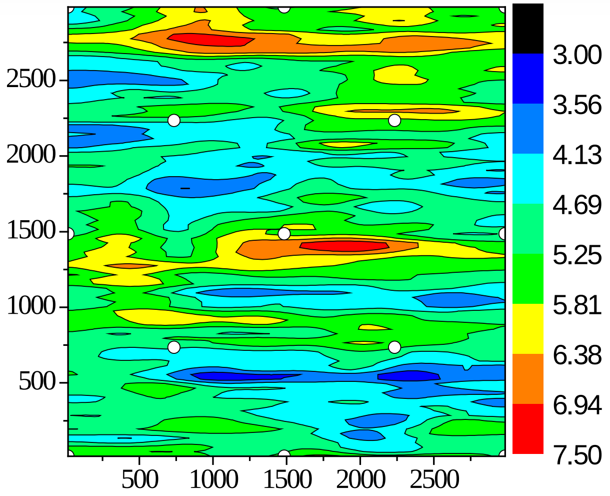
$$\eta_x / L_x = 0.5, \quad \eta_y / L_y = 0.05.$$



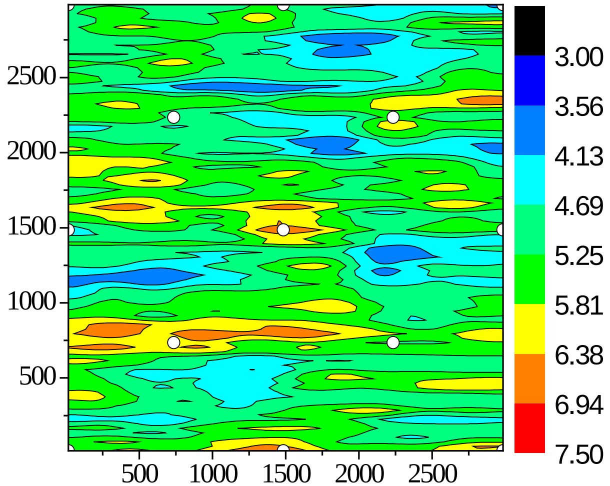
Initial setting

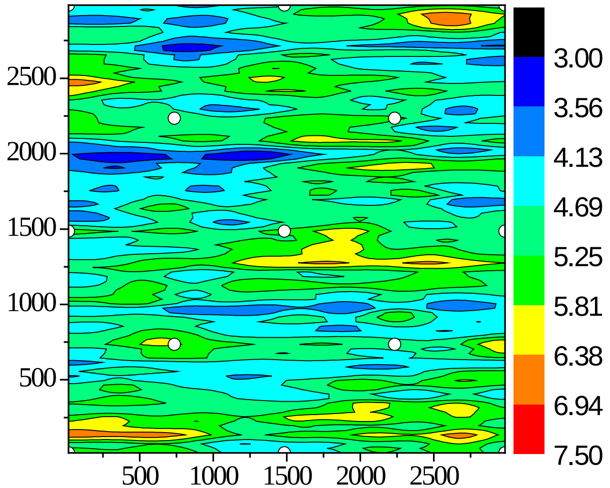
- Retained terms in KLE: 8×80
- Measurement error: 3%
- $N_e = 100$

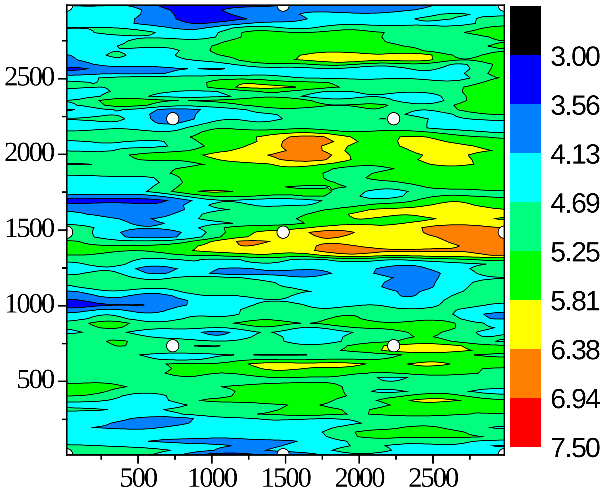
- Reference:



- Initial realizations by sampling of $\left\{\xi_n\right\}_{n=1}^{N}$:







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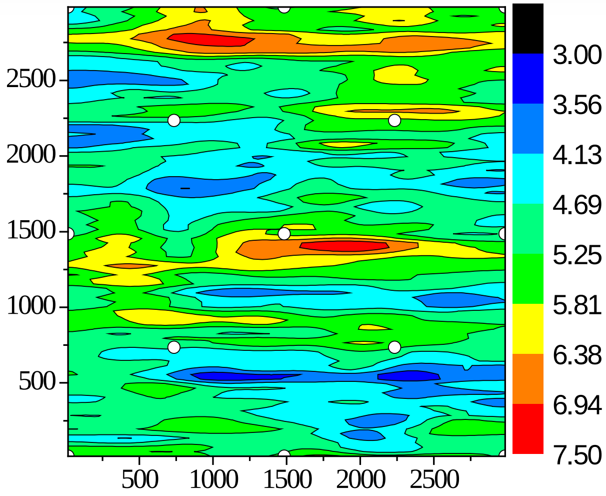
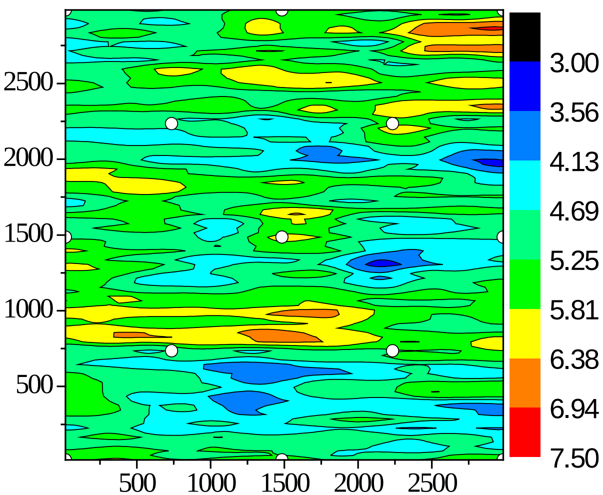
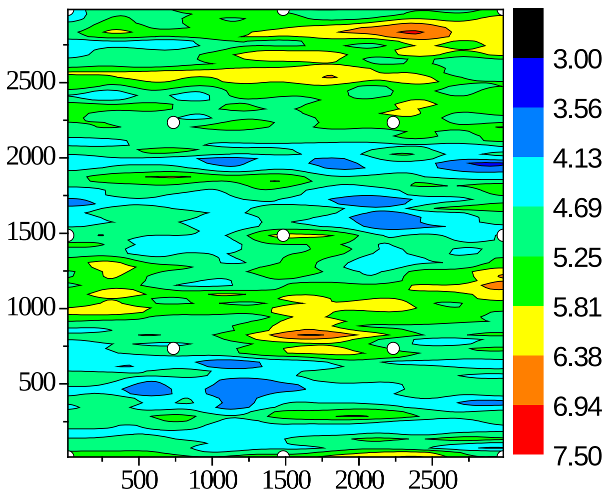
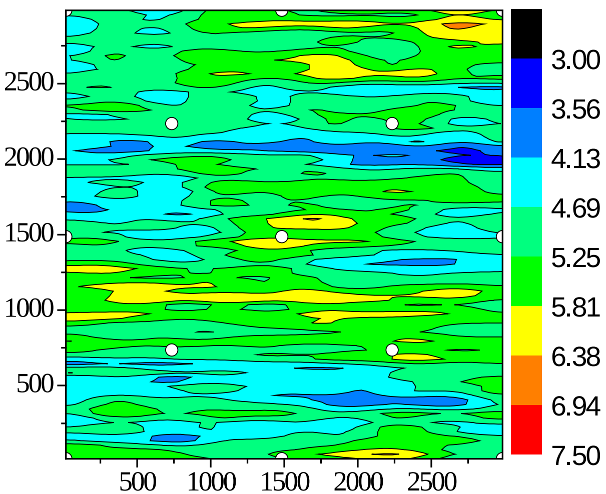


- Realizations from updated $\left\{\xi_n\right\}_{n=1}^{N}$:

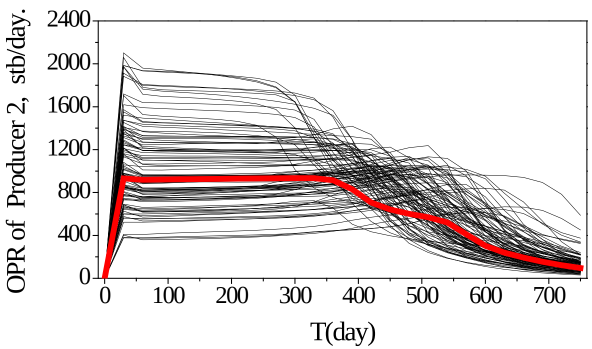


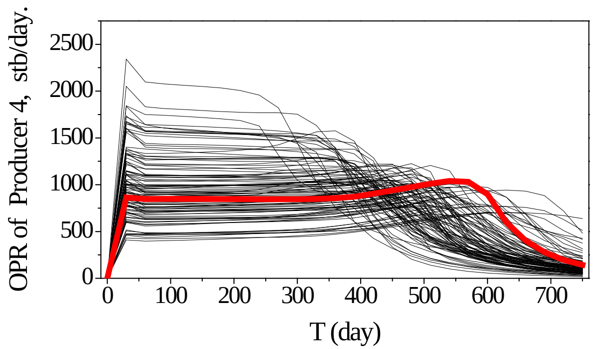


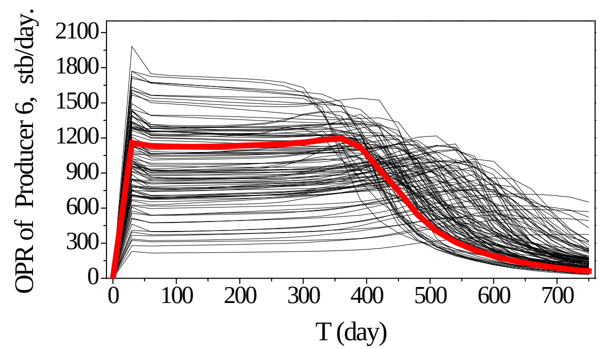


Data match

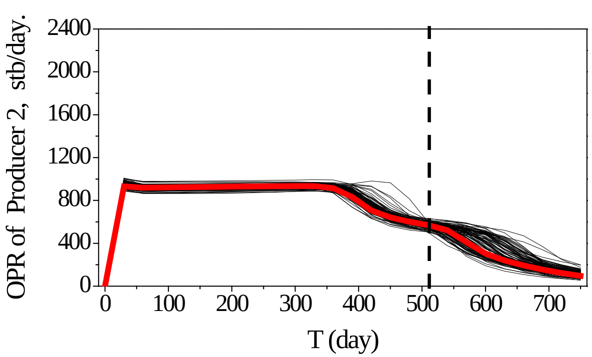
- initial realizations

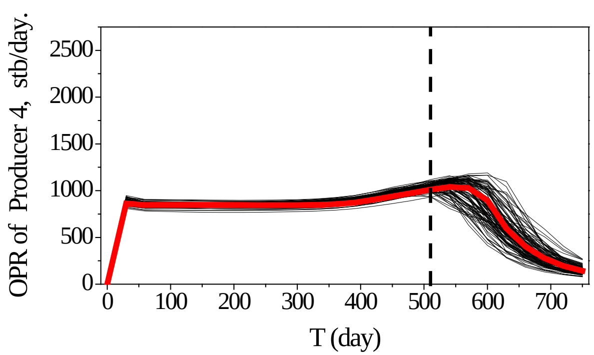


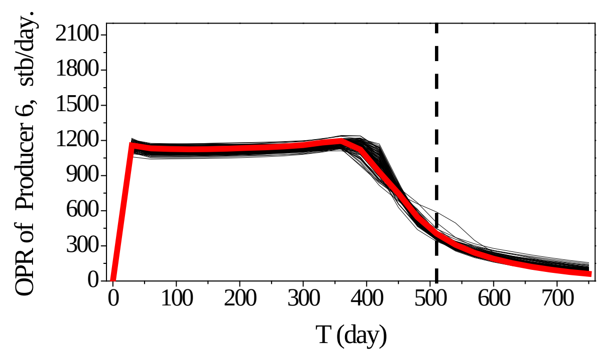




- updated realizations







Case 2: facies field estimation

Prior information:

- Proportion: 17%
- $k_f = 1000\text{mD}, \phi_f = 35\%$
- $k_n = 50\text{mD}, \phi_n = 15\%$
- Anisotropy direction: 30^o
- Mean length in major and minor correlation direction:

  1000ft, 120ft (approximately)

Facies: white shade

Note: this reference field is generated by SISIM program

Case 2: facies field estimation

$Y(\boldsymbol{x})$ for parameterizing facies

- $\langle Y \rangle = 0.0$, $\mathrm{var}(Y) = 0.5$
- Anisotropy direction: 30°
- Separable exponential covariance

$$C_Y(\boldsymbol{r}) = \sigma_Y^2 \exp\left[-\frac{|r_{major}|}{\eta_{major}} - \frac{|r_{minor}|}{\eta_{minor}} \right],$$

$$\eta_{major} = 1000\text{ft}, \quad \eta_{minor} = 120\text{ft}.$$

Other setting:

- Retained terms in KLE: 10×100
- Level set constant: 0.675

- Reference:

- Initial realizations by sampling of $\left\{\xi_n\right\}_{n=1}^{N}$:

- Reference:

- Realizations from updated $\left\{\xi_n\right\}_{n=1}^{N}$:

- Reference:

- Probabilistic maps: initial, step 3, final

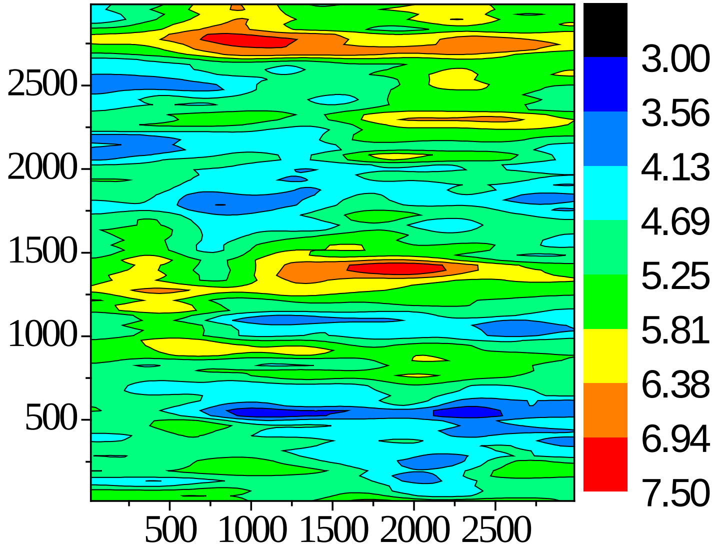
Uncertain anisotropy direction

- Model parameter:

$$\boldsymbol{m} = \left[\theta, \xi_1, \xi_2, \dots, \xi_N\right]^T$$

- Multi-facies

$$\boldsymbol{m} = \left[\theta^1, \dots, \theta^{n_f}, \boldsymbol{\xi}_n^1, \dots, \boldsymbol{\xi}_n^{n_f}\right]^T \text{ where } \boldsymbol{\xi}_n^i = (\xi_1^i, \xi_2^i, \dots, \xi_{N^i}^i)$$



Case 3: uncertain anisotropy direction

- True anisotropy direction: 30^{o}
- Initial statistics of θ:

  $U[18, 38]$

Note: here the directional correlation lengths are supposed to be known prior information.

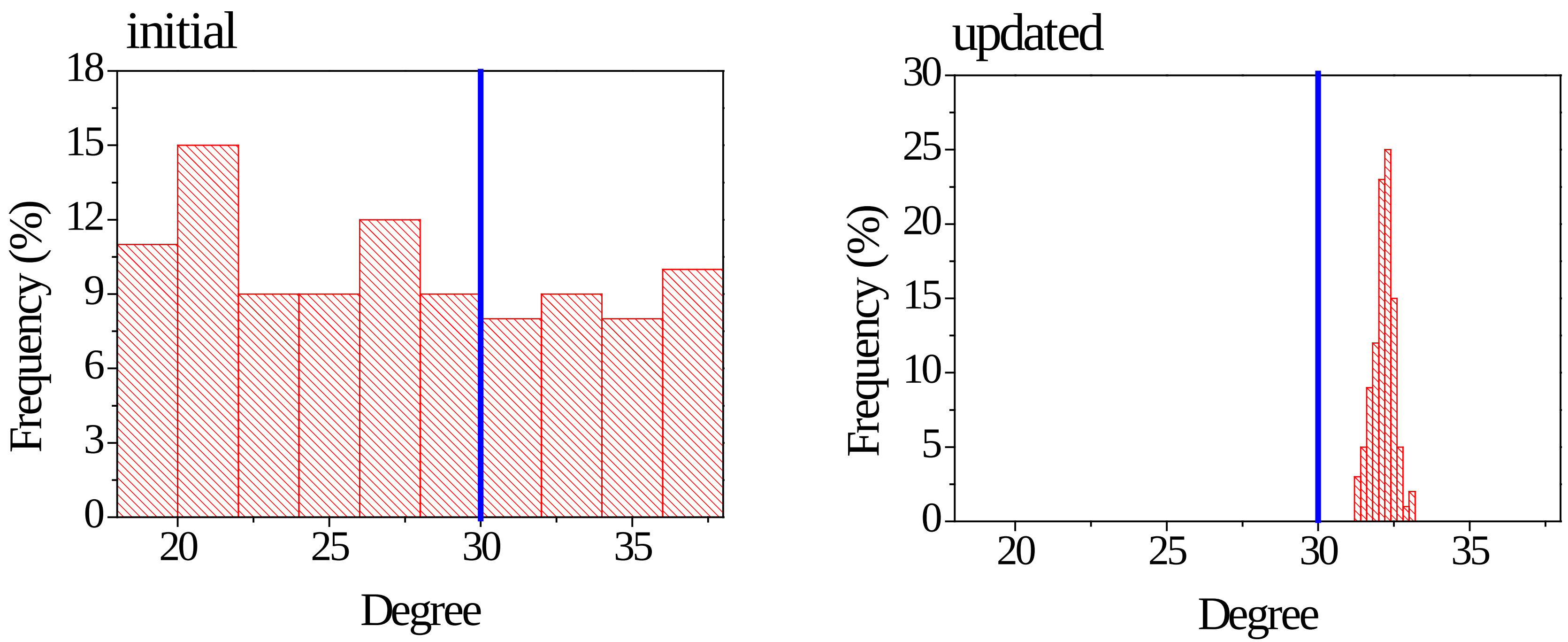
- Reference:

- Initial realizations by sampling of $\left\{\xi_n\right\}_{n=1}^{N}$ and θ:

- Reference:

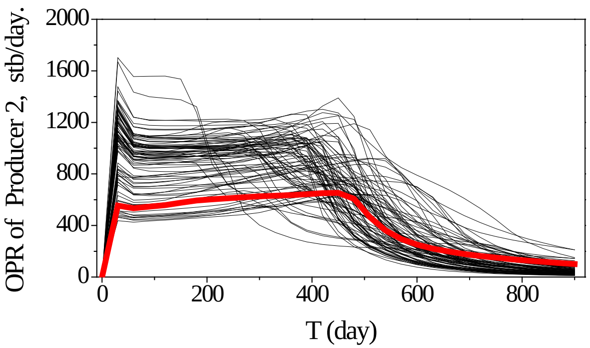
- Realizations from updated $\{\xi_n\}_{n=1}^{N}$ and θ :

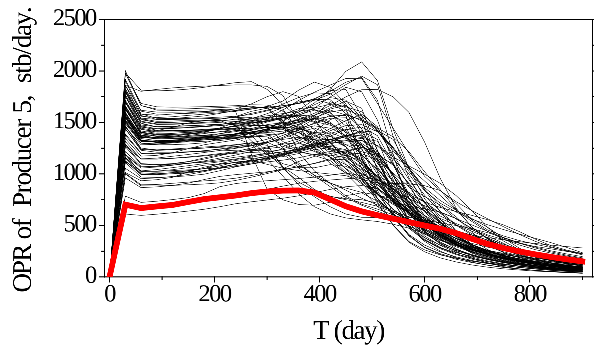
The ensemble of anisotropy direction

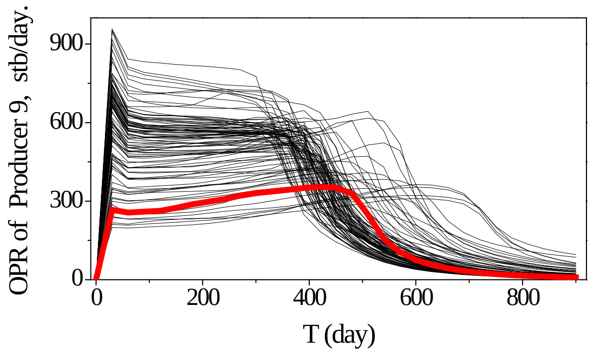


Data match

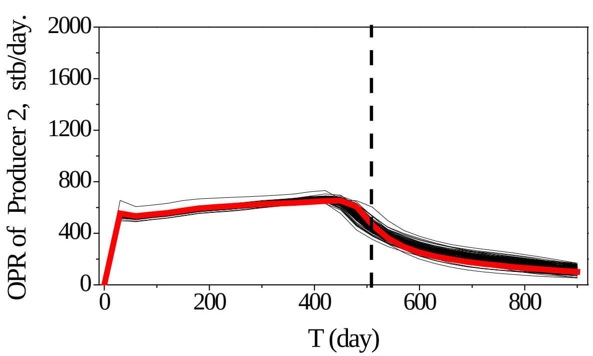
- initial realizations

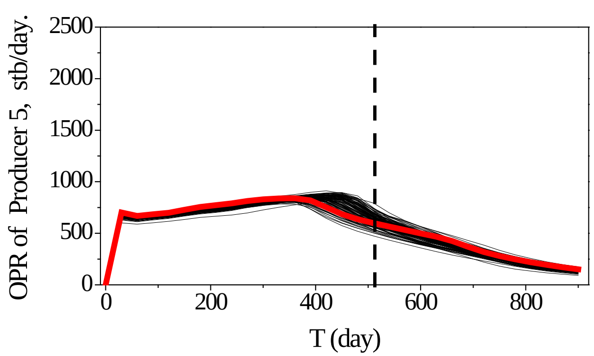


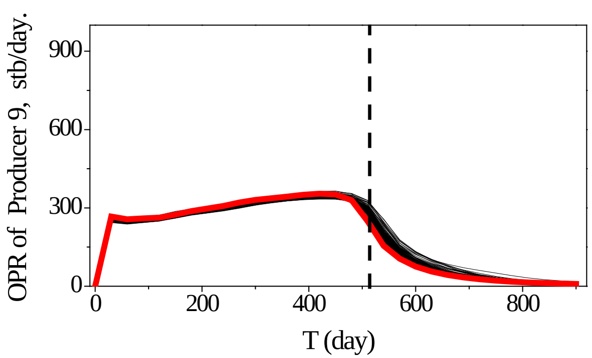




- updated realizations





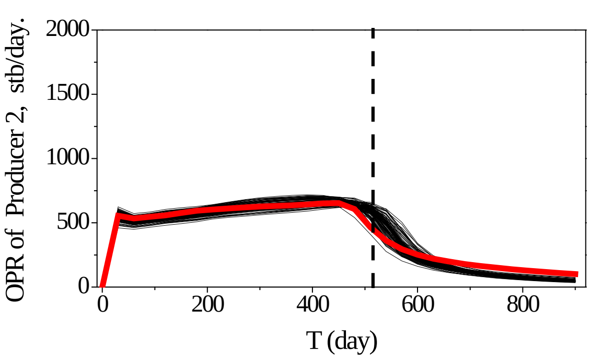


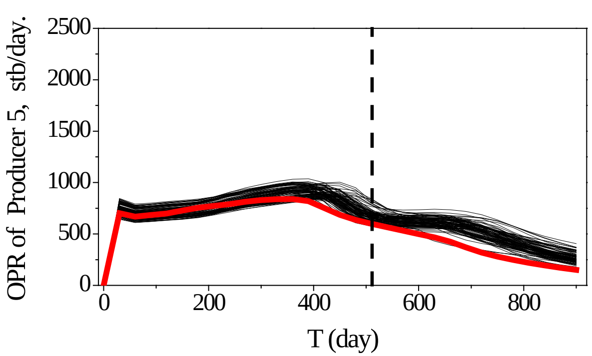
Without KLE based parameterization

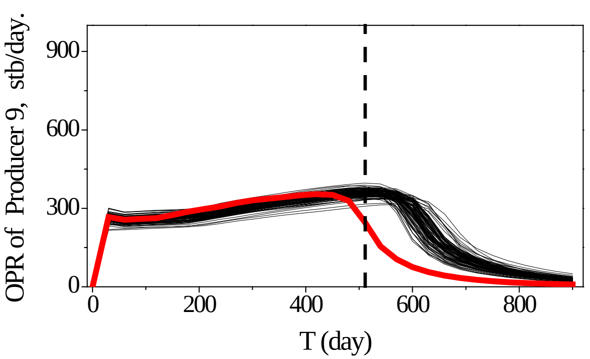
- Model parameter

$$\boldsymbol{m} = \left[Y(x_1,\omega), Y(x_2,\omega), \ldots, Y(x_{N_b},\omega) \right]^T .$$

where *Nb* denotes the number of grid blocks.







Uncertain correlation length

- Separable covariance for $Y(\boldsymbol{x},\omega)$

$$Y(\boldsymbol{x},\omega) \approx \bar{Y}(\boldsymbol{x}) + \sum_{n_x=1}^{N_x}\sum_{n_y=1}^{N_y} \xi_{n_x n_y}(\omega)\sqrt{\lambda_{n_x}\lambda_{n_y}/\sigma_Y^2}\, f_{n_x}(x)\, f_{n_y}(y)$$

$$= \bar{Y}(\boldsymbol{x}) + \sum_{n_x=1}^{N_x}\sum_{n_y=1}^{N_y} \xi_{n_x n_y}(\omega)\sqrt{\lambda_{n_x n_y}}\, f_{n_x n_y}(\boldsymbol{x})$$

- Model parameter:

$$\boldsymbol{m} = \left[\boldsymbol{\eta}^1, \ldots, \boldsymbol{\eta}^{n_f}, \boldsymbol{\xi}^1_{n_x n_y}, \ldots, \boldsymbol{\xi}^{n_f}_{n_x n_y}\right]^T$$

where

$$\boldsymbol{\eta}^i = (\eta_x^i, \eta_y^i)$$

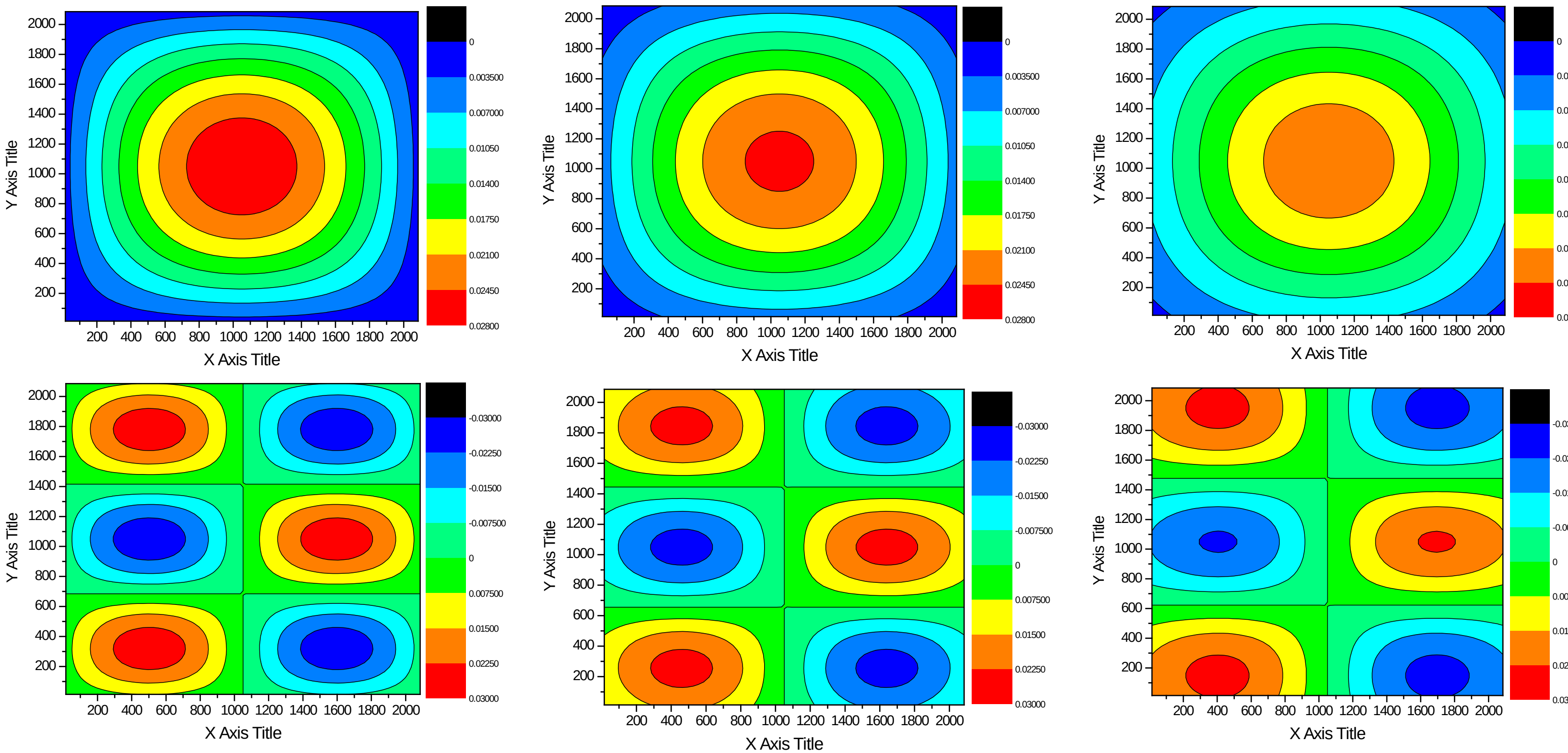
$$\boldsymbol{\xi}^i_{n_x n_y} = (\xi^i_{11}, .., \xi^i_{1N_y^i}, \ldots, \xi^i_{N_x^i 1}, .., \xi^i_{N_x^i N_y^i})$$

The advantages of using separable covariance model

- Low computational burden for solving eigenvalue problem
- Each eigenfunction has similar profile with respect to different correlation length

$f_{11}(\boldsymbol{x})$

$f_{23}(\boldsymbol{x})$



$\eta_x / L_x = \eta_y / L_y = 0.05$ 0.15 0.25

Case 4: uncertain directional mean length

Prior information:

- Proportion: 25%
- $k_f = 1000\text{mD}, \phi_f = 25\%$
- $k_n = 50\text{mD}, \phi_n = 15\%$

Facies: white shade
Normal medium: black shade

Case 4

$Y(\boldsymbol{x})$ for parameterizing facies

- $\langle Y \rangle = 0.0$, $\mathrm{var}(Y) = 0.5$
- Gaussian covariance

$$C_Y(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sigma_Y^2 \exp\left[-\left(\frac{|x_1 - x_2|}{\eta_x} \right)^2 - \left(\frac{|y_1 - y_2|}{\eta_y} \right)^2 \right].$$

- Reference: $\eta_x / L_x = 0.2,\ \ \eta_y / L_y = 0.08$

Other setting:

- Initial statistics:

$$\eta_x / L_x \sim U(0.05, 0.25), \quad \eta_y / L_y \sim U(0.05, 0.25)$$

- Retained terms in KLE: 18×18
- Level set constant: 0.477

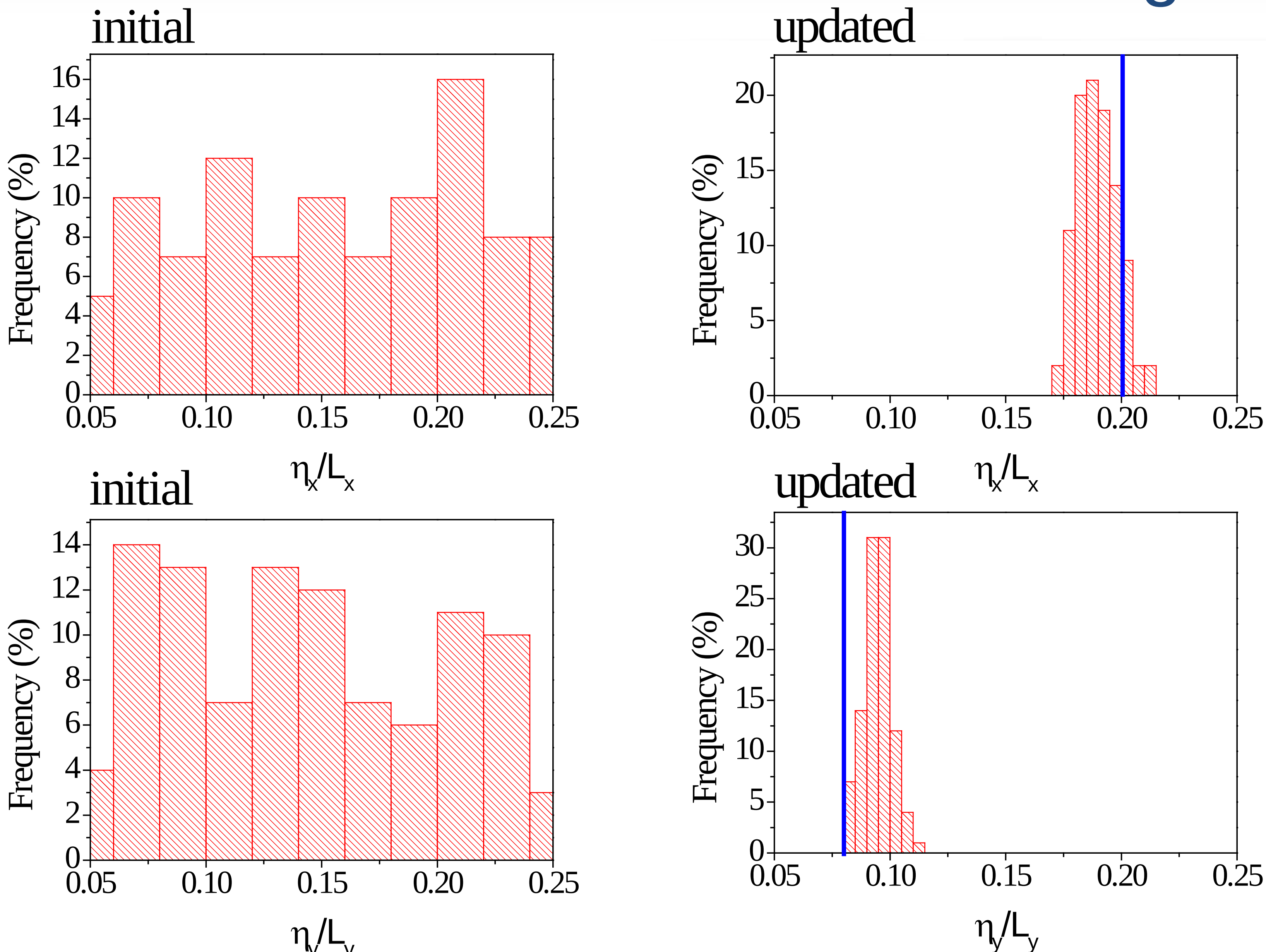
- Reference:

- Initial realizations by sampling of $\left\{\xi_{n_x n_y}\right\}$ and η_x, η_y :

- Reference:

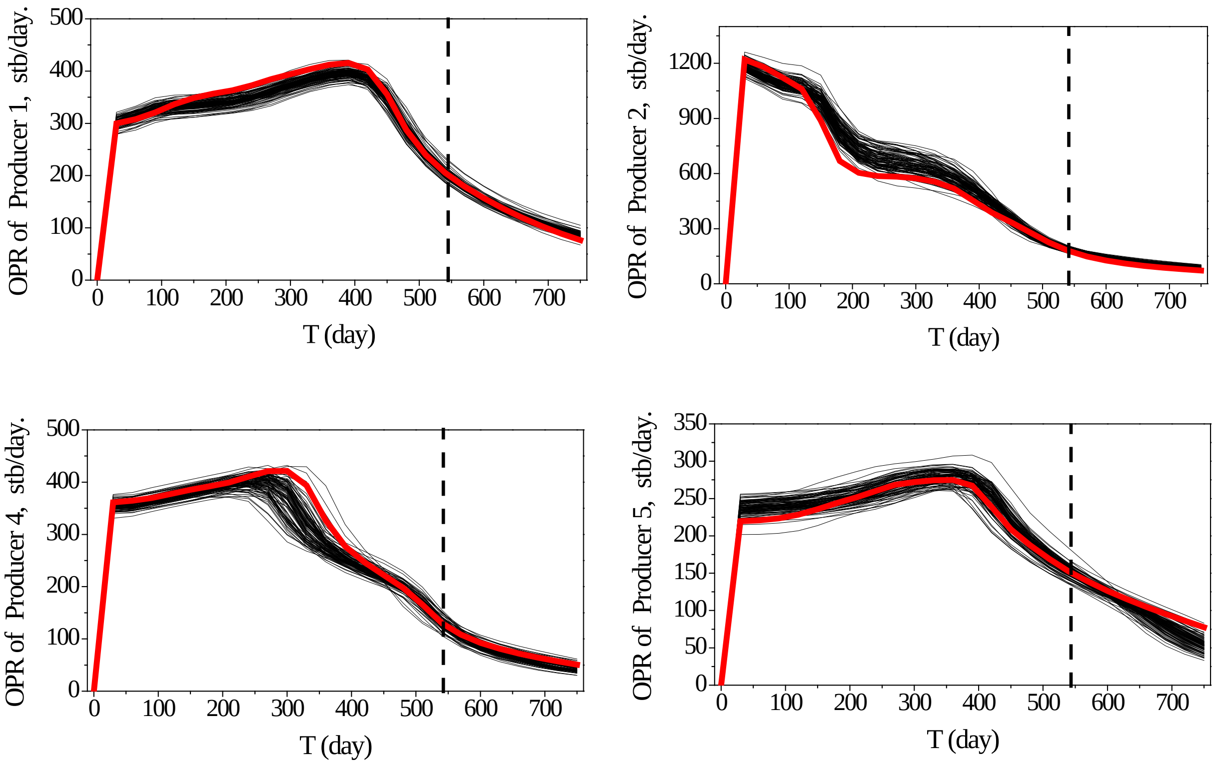
- Probabilistic maps: initial, step 3, final

The ensemble of correlation length



Data match

- updated realizations



Case 5: uncertain facies mean size

Prior information:

- Proportion: 20%
- $k_{f_1} = 2000\text{mD}; \; k_{f_2} = 1\text{mD}; \; k_n = 100\text{mD}$
- $\phi_{f_1} = 30\%; \; \phi_{f_2} = 10\%; \; \phi_n = 20\%;$

Facies 1: white shade
Facies 2: black shade
Normal medium: gray shade

Case 5

$Y(\boldsymbol{x})$ for parameterizing facies

- $<Y^1>= 0.0,\ \sigma^2_{Y^1} = 0.5; <Y^2>= 0.0,\ \sigma^2_{Y^2} = 0.5$
- Gaussian covariance

$$C_{Y^1}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sigma^2_{Y^1} \exp\left[-\left(\frac{|x_1 - x_2|}{\eta^1} \right)^2 - \left(\frac{|y_1 - y_2|}{\eta^1} \right)^2 \right], \quad C_{Y^2}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sigma^2_{Y^2} \exp\left[-\left(\frac{|x_1 - x_2|}{\eta^2} \right)^2 - \left(\frac{|y_1 - y_2|}{\eta^2} \right)^2 \right]$$

- Reference: $\eta^1 / L = 0.2,\ \ \eta^2 / L = 0.08$

Other setting:

- Initial statistics:

$$\eta^1 / L \sim U(0.05, 0.25), \quad \eta^2 / L \sim U(0.05, 0.25)$$

- Retained terms in KLE: 18×18
- Level set constants: 0.595; 0.477

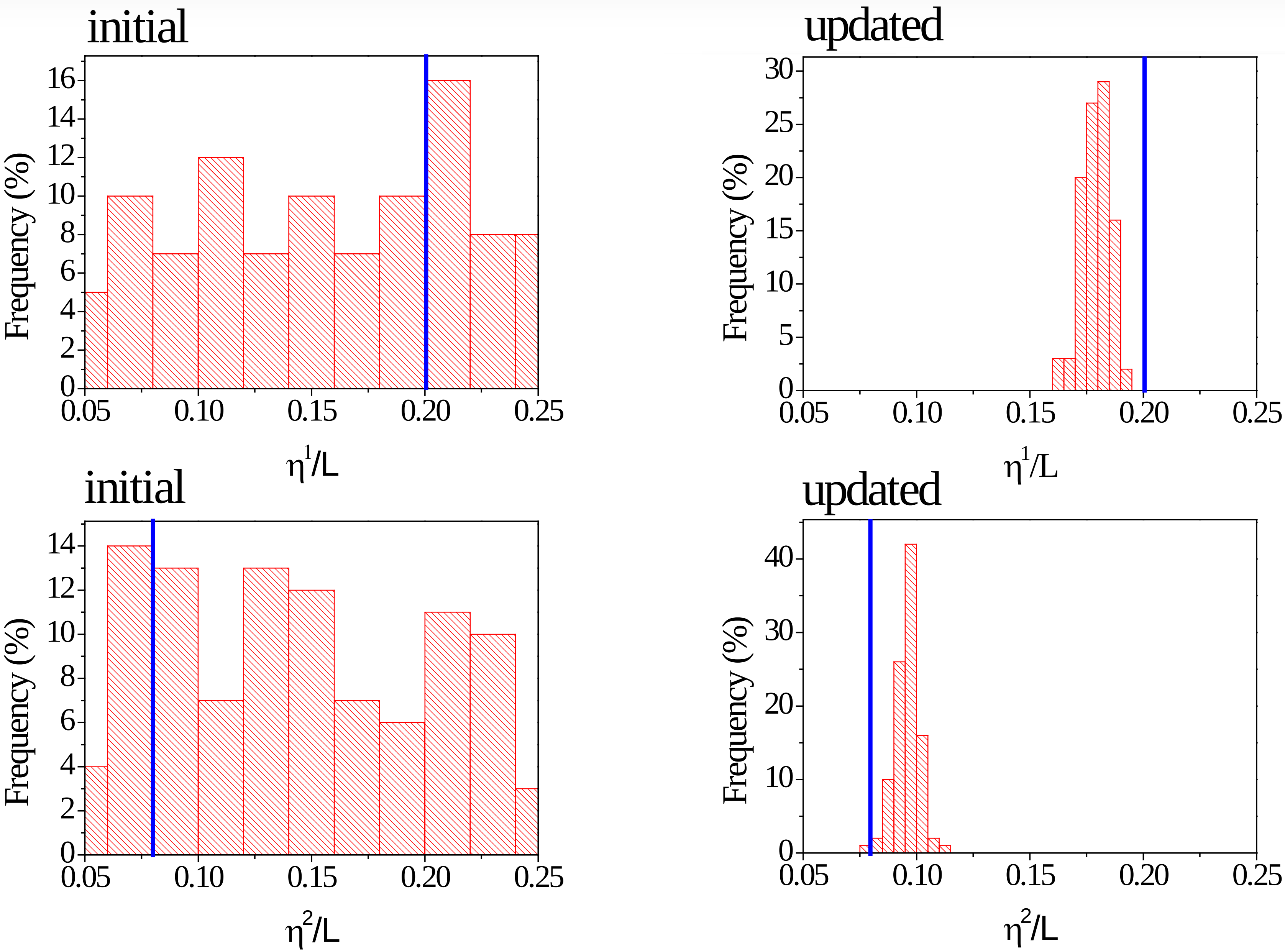
- Reference:

- Initial realizations by sampling of $\left\{\xi^{1}_{n_x n_y}\right\}, \left\{\xi^{2}_{n_x n_y}\right\}$ and η^{1}, η^{2}:

Probabilistic maps

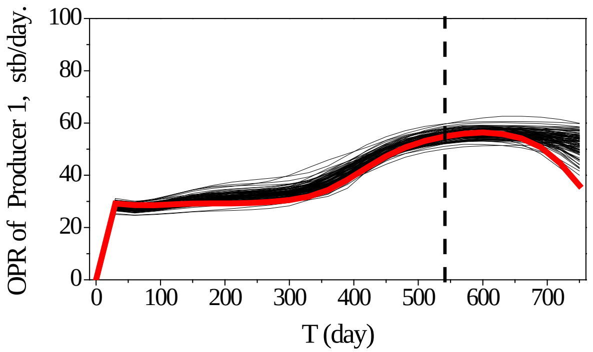
reference initial updated

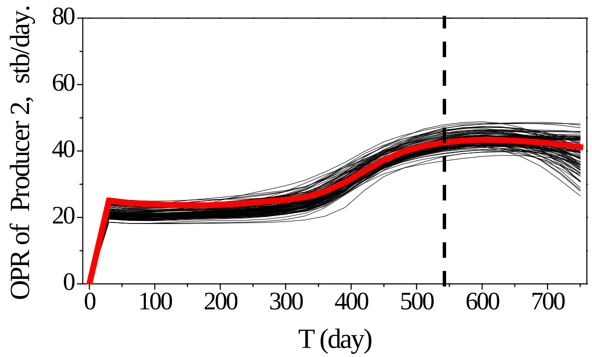
The ensemble of correlation length

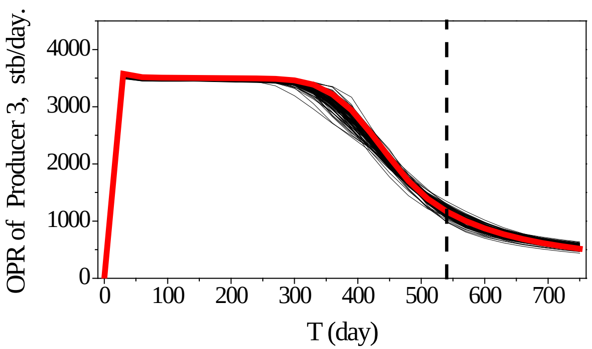


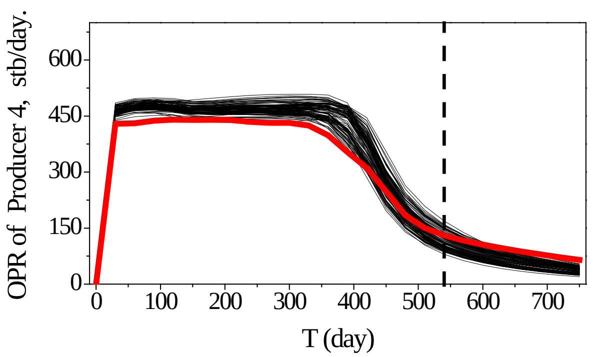
Data match

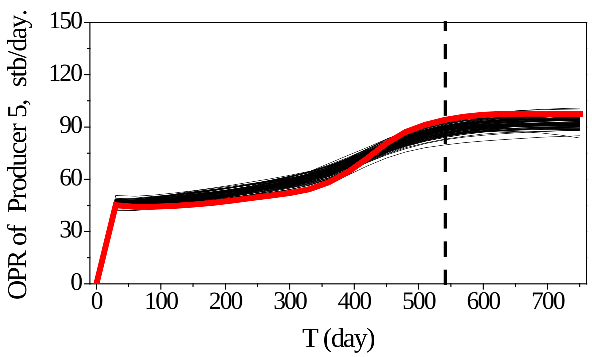
- updated realizations











Summary

- The KL expansion is a proper global parameterization technique to preserve the geostatistical characteristics of a Gaussian parameter field in the updating process of the history matching method;

- By using KL expansion, the correlation parameters, such as anisotropy direction and correlation length, can be explicitly parameterized and updated;

- In this work, the proportion of each facies is supposed to be known. If this prior information is uncertain, we can treat the level set constants in the parameterization scheme as model parameters for taking account of this uncertainty.

Summary

- The KL expansion based global parameterization technique is only suitable for parameterizing Gaussian random fields (that can be completely depicted by the first two moments). For non-Gaussian random fields, such as curvilinear channels or fractures, other parameterization techniques should be investigated;

- In this work, although we only use two dimensional cases to test the proposed method, the KL expansion based method can be easily extended to three dimensional problems.

Thanks!